CycLiq

Model Description

A brief description of the model is presented here, readers should refer to Wang et al. (2014) for the full formulation of the original model.

The basic equations for the multiaxial model are:

$$\dot{\varepsilon}_v^{\,e} = \frac{\dot{p}}{K}; \qquad \dot{\mathbf{e}}^e = \frac{\dot{\mathbf{s}}}{2G} \tag{1}$$

$$\dot{\varepsilon}_v^p = \langle L \rangle D; \qquad \dot{\mathbf{e}}^p = \langle L \rangle \mathbf{m} \tag{2}$$

$p = \mathrm{tr}(\boldsymbol{\sigma})/3$ is the mean effective stress, with $\boldsymbol{\sigma}$ being the effective stress tensor; $\mathbf{s} = \boldsymbol{\sigma} - p\mathbf{I}$ is the deviatoric stress, $\mathbf{I}$ being the rank two identity tensor; $\varepsilon_v = \mathrm{tr}(\boldsymbol{\varepsilon})$ is the volumetric strain, $\boldsymbol{\varepsilon}$ being the strain tensor; $\mathbf{e} = \boldsymbol{\varepsilon} - \varepsilon_v / 3\mathbf{I}$ is the deviatoric strain tensor. L is the plastic loading index and $\mathbf{m}$ the deviatoric strain flow direction. The deviatoric stress ratio tensor is here defined as $\mathbf{r} = \frac{\mathbf{s}}{p}$, and $q = \sqrt{\frac{3}{2}\mathbf{s}:\mathbf{s}}$, $\eta = \frac{q}{p}$.

The total stress-strain relation can be formulated by combining Eqs. (1) and (2) to be:

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{2G} p\dot{\mathbf{r}} + (\frac{1}{2G}\mathbf{r} + \frac{1}{3K}\mathbf{I})\dot{p} + (\mathbf{m} + \frac{D}{3}\mathbf{I})\langle L \rangle \tag{3}$$

with the elastic moduli G and K defined as suggested by Richart et al. (1970):

$$G = G_o \frac{(2.973 - e_{in})^2}{1 + e_{in}} p_a \left(\frac{p}{p_a}\right)^{\frac{1}{2}} \tag{4}$$

$$K = \frac{1 + e_{in}}{\kappa} p_a \left(\frac{p}{p_a}\right)^{\frac{1}{2}} \tag{5}$$

The critical, maximum stress ratio and reversible dilatancy surfaces are shown schematically in Fig. 1.

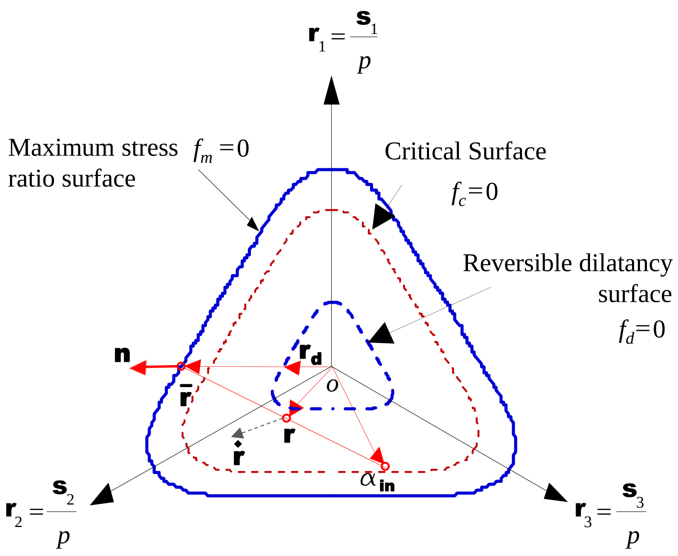

Fig. 1. Schematic illustration of critical state, maximum stress ratio and reversible dilatancy surfaces with mapping rules.

The function $g(\theta)$ in this model is modified based on Zhang's (1997) original proposition:

$$g(\theta) = \left(\frac{1}{1+M_p(1+\sin 3\theta - \cos^2 3\theta)/6 + (M_p - M_{p,o})\cos^2 3\theta / M_{p,o}}\right) \quad (6)$$

$$M_p = \frac{6\sin\phi_f}{3-\sin\phi_f} \quad (7)$$

$$M_{p,o} = \frac{2\sqrt{3}\tan\phi_f}{\sqrt{3+4\tan^2\phi_f}} \quad (8)$$

$M_p = M\exp(-n^b\Psi)$ is the peak mobilized stress ratio at triaxial compression and ϕ_f is the corresponding friction angle, $M_{p,o}$ is the peak mobilized stress ratio under torsional shear after isotropic consolidation. The state parameter Ψ proposed by Been and Jefferies (1985) is introduced to consider the dependency of sand behaviour on the current state.

Plastic loading is determined in three dimensional space by the load index L:

$$L = \frac{\mathbf{L} : \dot{\boldsymbol{\sigma}}}{H} = \frac{p\dot{\mathbf{r}} : \mathbf{n}}{H} \tag{9}$$

Here $\mathbf{n}$ is a unit deviatoric tensor serving as the loading direction in deviatoric stress space in the model, and the loading direction $\mathbf{L}$ is defined as $\mathbf{L} = \mathbf{n} - \frac{1}{3}(\mathbf{n} : \mathbf{r})\mathbf{I}$. Plastic loading is induced when $L > 0$, and load reversal occurs at $L < 0$.

It is further assumed that the deviatoric strain flow direction $\mathbf{m}$ in Eq. (3) is the same as the loading direction in deviatoric stress space so as:

$$\mathbf{m} = \mathbf{n} = \bar{\mathbf{r}} / \sqrt{\bar{\mathbf{r}} : \bar{\mathbf{r}}} \tag{10}$$

Here $\bar{\mathbf{r}}$ represents the projection of the current stress point on the maximum stress ratio surface in deviatoric stress space (Fig. 1). The projection of current stress ratio on the maximum stress ratio surface $\bar{\mathbf{r}}$ is defined as the intersection between the extension of the line from the previous load reversal point $\boldsymbol{\alpha}_{\mathbf{in}}$ to $\mathbf{r}$ and the maximum stress ratio surface:

$$\bar{\mathbf{r}} = \boldsymbol{\alpha}_{\mathbf{in}} + \beta(\mathbf{r} - \boldsymbol{\alpha}_{\mathbf{in}}) \tag{11}$$

When the loading index *L* is positive, plastic loading occurs. Once *L* becomes negative, load reversal takes place and the projection centre $\boldsymbol{\alpha}_{\mathbf{in}}$ is updated to be the current stress ratio.

The plastic modulus *H* can then be defined as:

$$H = \frac{2}{3} hG \exp(-n^p \Psi) \left(\frac{M \exp(-n^b \Psi)}{M_m} \left(\frac{\bar{\rho}}{\rho} \right) - 1 \right) \tag{12}$$

where $\bar{\rho}$ is the distance between $\bar{\mathbf{r}}$ and $\boldsymbol{\alpha}_{\mathbf{in}}$, and ρ the distance between $\mathbf{r}$ and $\boldsymbol{\alpha}_{\mathbf{in}}$.

The mapping rule for reversible dilatancy is defined so that the projection of the current stress ratio on the reversible dilatancy surface $\mathbf{r_d}$ is the intersection between $\bar{\mathbf{r}}$ and the reversible dilatancy surface:

$$\mathbf{r_d} = \frac{M_d}{M_m}\bar{\mathbf{r}} = \frac{M\exp(n^d\Psi)}{M_m}\bar{\mathbf{r}} \tag{13}$$

According to the propositions made by Shamoto et al. (1997) and Zhang (1997), the dilatancy of sand is decomposed into a reversible and an irreversible component, through which the dilatancy during load reversal and cyclic loading can be properly reflected. In this model, the dilatancy rate D is determined by combining the reversible part D_{re} and irreversible part D_{ir}: The generation and release of reversible dilatancy can then be judged by the angle between $\mathbf{r_d} - \mathbf{r}$ and $\mathbf{n}$:

$$D_{re} = \frac{\dot{\varepsilon}_{vd,re}}{\dot{\gamma}^p} = \begin{cases} D_{re,gen}, & (\mathbf{r_d} - \mathbf{r}) : \mathbf{n} < 0 \\ D_{re,rel}, & (\mathbf{r_d} - \mathbf{r}) : \mathbf{n} > 0 \end{cases} \tag{14}$$

The generation rate of reversible dilatancy is:

$$D_{re,gen} = \sqrt{\frac{2}{3}} d_{re,1} \left(\mathbf{r_d} - \mathbf{r}\right) : \mathbf{n} \tag{15}$$

Reversible dilatancy remains non-positive and is released after load reversal, the release rate is defined as:

$$D_{re,rel} = \left(d_{re,2}\chi\right)^2 / p \tag{16}$$

$d_{re,2}$ is another dilatancy parameter used to calculate the release of reversible dilatancy. $\chi = \min(-d_{ir}\frac{\varepsilon_{vd,re}}{\varepsilon_{vd,ir}^{pr}}, 1)$ is a function controlling the reversible dilatancy release process, where d_{ir} is an irreversible dilatancy constant and $\varepsilon_{vd,ir}^{pr}$ is the $\varepsilon_{vd,ir}$ at previous load reversal.

Irreversible dilatancy rate D_{ir} defined as:

$$D_{ir} = \frac{\dot{\varepsilon}_{vd,ir}}{\left|\dot{\varepsilon}_q^p\right|}$$
$$= d_{ir} \exp(n^d \Psi - \alpha \varepsilon_{vd,ir})(< M_d - \eta > \exp(\chi) + \left(\frac{\gamma_{d,r} < 1 - \exp(n^d \Psi) >}{\gamma_{d,r} < 1 - \exp(n^d \Psi) > + \gamma_{mono}} \right)^2) \tag{17}$$

Here α is a parameter controlling the decrease rate of irreversible dilatancy, γ_{mono} is the shear strain since the last stress reversal and $\gamma_{d,r}$ is a reference shear strain. $\langle \; \rangle$ are the MacCauley backets that yield $\langle x \rangle = x$ if $x > 0$ and $\langle x \rangle = 0$ if $x \le 0$. The $\exp(n^d \Psi - \alpha \varepsilon_{vd,ir})$ part of the equation reflects asymptotic accumulation of irreversible dilatancy, and the part $\left(\frac{\gamma_{d,r} < 1 - \exp(n^d \Psi) >}{\gamma_{d,r} < 1 - \exp(n^d \Psi) > + \gamma_{mono}} \right)^2$ reflects the decreasing dilatancy rate during each monotonic loading process.

Model Parameters

The model parameters for Toyoura sand (calibrated by Zou et al., 2018) are listed in Table 1. Note the parameter $\gamma_{d,r}$ is kept at a default value of 0.05.

Table 1. Model parameters for the simulations.

Sand	G_o	κ	h	M	$d_{re,1}$	$d_{re,2}$	d_{ir}	α	$\gamma_{d,r}$	n^p	n^d	λ_c	e_0	ξ
Toyoura sand	200	0.008	1.8	1.35	0.35	30	0.75	10	0.05	1.1	7.8	0.019	0.934	0.7

Calibration Method

The calibration method for some parameters have been documented by previous researchers, including the elastic modulus constants (G_0, κ), plastic modulus parameter (h) and critical state parameters (M, λ_c, e_0, ξ).

The state parameter constants n^p and n^d can be determined through $n^p = \ln(M/\eta_p)/\Psi_p$ and $n^d = \ln(M_d/M)/\Psi_d$, where η_p and Ψ_p are η and Ψ at peak stress ratio in a monotonic drained triaxial test, and M_d and Ψ_d are those at reversible dilatancy sign change points.

Drained cyclic torsional or triaxial tests should be used for the determination of n^d here, as M_d can only be acquired once irreversible dilatancy is negligible after a number of loading cycles. The reversible dilatancy parameters $d_{re,1}$ can be determined using the relationship between η and $\frac{d\varepsilon_{vd}}{d\gamma^p}$ from drained cyclic tests as suggested by Zhang and Wang (2012), and $d_{re,2}$ should then be chosen to ensure the release of reversible dilatancy.

For the irreversible dilatancy parameters (d_{ir} and α especially), a trial-and-error process should be adopted to simulate the stress strain behaviour of undrained cyclic torsional/triaxial tests of different initial confining pressure or shear stress amplitude, as was described by Zhang and Wang (2012). The parameter d_{ir} mainly determines how fast liquefaction is reached in undrained cyclic tests, and α controls the decrease rate of irreversible dilatancy.

Example input file

This section presents the input file for FLAC3D v5.0 for the simulation of a undrained cyclic torsional shear test. The FLAC3D simulation is carried out using a single zone of unit dimensions (1m x 1m x 1m). The gridpoints are fixed and the shear strain is applied as horizontal displacement at the upper gridpoints. A servo function is used in order to perform cyclic testing under constant stress amplitude (τ_{cyc} =25kPa). Initial stress field corresponds to vertical effective stress equal to 100kPa and horizontal effective stress equal to 100kPa (i.e. lateral earth pressure coefficient at rest equal to k_o =1).

Table 2. Input file for undrained cyclic torsional shear test simulation.

Parameters	e_{in}	poros	Dens ($\times 10^3$kg/m^3)	τ_{cyc} (kPa)	S_{zz0} (kPa)	K_0
Values	0.773	0.435	1.497	25	100	1.0

```
new
title "Undrained cyclic shear test"
config fluid
config cppudm

model load modelPost_sandliq005_64__v190122_.dll
gen zone brick size 1 1 1 p0 0 0 0 p1 1.0 0 0 p2 0 1.0 0 p3 0 0 1.0 group cycliq_p1

model CycliqCPSP
model fluid fl_iso
set fluid off

prop dens 1.497 poros 0.435 ein1=0.773 ;Toyoura sand
prop G01=200. kappa1=0.008 h1=1.8 Mc1=1.35 dre11=0.35e0 dre21=30.e0 dir1=0.75e0 ...
eta1=10. rdr1=0.05 nb1=1.1 nd1=7.8 lamdac1=0.019 e01=0.934 ksi1=0.7 ...
; Toyoura sand model parameters

ini fmod 2.e6 fden 1.000

;---------------- Initial Conditions -------------------------
[xv_ = 0.5e-5 ]
[sigma3_ = -100.0 ]
[sxy_max_ = 25.0 ]
[sxy_min_ = -25.0 ]
[sxy0_ = (sxy_max_+sxy_min_)/2.0 ]
```

```
fix x y z
ini syy @sigma3_
ini sxx @sigma3_
ini szz @sigma3_
ini sxy @sxy0_
ini sxz 0.
ini syz 0.

prop mElastFlag1=2 ; 2-elastic (mElastFlag1 - elastic and plastic flag)
solve

prop mElastFlag1=1 ; 1-plastic (mElastFlag1 - elastic and plastic flag)
ini xdis 0 ydis 0 zdis 0 xvel 0 yvel 0 zvel 0 ;clear displacement and velocity

;----------------hist variables------------------------------
define constants
global pzCL_a = z_near(0.5,0.5,0.5);
global pgp1CL_a = gp_near( 0.0, 0.0, 1.0)
global pgp2CL_a = gp_near( 0.0, 1.0, 1.0)
global pgp3CL_a = gp_near( 1.0, 0.0, 1.0)
global pgp4CL_a = gp_near( 1.0, 1.0, 1.0)
end
@constants

def p_1
local tmp = 1.0/3.0 *(z_sxx(pzCL_a) + z_syy(pzCL_a) + z_szz(pzCL_a))
global p_1 = (tmp + z_pp(pzCL_a))*(-1.0)
global q_1 = z_sxy(pzCL_a)
global settlement_1 = -1.0/ 4.0 * (gp_zdisp(pgp1CL_a) + gp_zdisp(pgp2CL_a) +
                                gp_zdisp(pgp3CL_a) + gp_zdisp(pgp4CL_a))
global strain_a_1 = settlement_1 / 1.0
global szz_e_1 = ( z_szz(pzCL_a) + z_pp(pzCL_a))*(-1.0)
global sxx_e_1 = (z_sxx(pzCL_a) + z_pp(pzCL_a)) *(-1.0)
global szz_t_1 = z_szz(pzCL_a) *(-1.0)
global sxx_t_1 = z_sxx(pzCL_a) *(-1.0)
end

hist id 2 fish @p_1
hist id 3 fish @q_1
his id 201 zone sxx 0.5 0.5 0.5
his id 202 zone syy 0.5 0.5 0.5
his id 203 zone szz 0.5 0.5 0.5

his id 204 zone sxy 0.5 0.5 0.5
```

```
his id 205 zone sxz 0.5 0.5 0.5
his id 206 zone syz 0.5 0.5 0.5

his id 207 gp xd 0.,1.,0.
his id 208 gp xv 0. 1. 0.

hist nstep 2

;--------------- Cyclic Loading -------------------------------
range name x0 x -0.1 0.1
range name x1 x 0.9 1.1
range name y0 y -0.1 0.1
range name y1 y 0.9 1.1
range name z0 z -0.1 0.1
range name z1 z 0.9 1.1

[xv_pos_ = xv_ ]
[xv_neg_ = -1.0*xv_ ]

ini xv @xv_pos_ range y1

def gamma_contl_(gam_)
p_z=z_near(0.5,0.5,0.5)
gp_pnt_ = gp_near( 0.0e+00, 1.0e+00, 0.0e+00)
global gamma_ = abs(gp_xdisp(gp_pnt_))
global gamma_max_ = gam_

loop while gamma_ < gamma_max_
command
step 1
endcommand

if z_sxy(p_z)>sxy_max_ then
command
ini xv @xv_neg_ range y1
endcommand
endif

if z_sxy(p_z)<sxy_min_ then
command
ini xv @xv_pos_ range y1
endcommand
endif
gamma_ = abs(gp_xdisp(gp_pnt_))
```

```
endloop

global sgn_z_sxy_ = sgn( z_sxy(p_z))
global z_xy_0_ = z_sxy(p_z)

loop while z_sxy(p_z) * sgn_z_sxy_ > 0.0 ; back to zero shear stress state ...
; after cyclic loading
command
step 1
endcommand

if sgn_z_sxy_>0.0 then
command
ini xv @xv_neg_ range y1
endcommand
endif

if sgn_z_sxy_ <0.0 then
command
ini xv @xv_pos_ range y1
endcommand
endif
endloop
end

@gamma_contl_(0.10)

;----------plot simulation results----------------------------
plot create plot 'sxy_p'
plot set job on
plot set viewtitle on
plot set viewtitle text 'plot_sxy_p'
plot add hist 3 linestyle style solid color red width 2 &
vs 2

plot create plot 'sxy_gamma'
plot set job on
plot set viewtitle on
plot set viewtitle text 'plot_sxy_gamma'
plot add hist 3 linestyle style solid color red width 2 &
vs 207

plot create plot 'p_gamma'
plot set job on
```

```
plot set viewtitle on
plot set viewtitle text 'plot_p_gamma'
plot add hist -2 linestyle style solid color red width 2 &
vs 207

plot create plot 'xv'
plot set job on
plot set viewtitle on
plot set viewtitle text 'xv'
plot add hist 208 linestyle style solid color blue width 2

return
```