

Updates to a Practice-Oriented Liquefaction Model

E. M. Dawson¹ and L. H. Mejia²

¹URS Corporation, 915 Wilshire Blvd., Suite 700, Los Angeles, CA. 90017; email: ethan_dawson@urscorp.com

²URS Corporation, 1333 Broadway, Suite 800, Oakland CA, 94612; email: lelio_mejia@urscorp.com

ABSTRACT

Updates are described of a practice-oriented, cycle-counting liquefaction model that has been in use since 1985. The model consists of a linear-elastic perfectly-plastic Mohr-Coulomb model, coupled with an empirical pore pressure generation procedure. Excess pore pressure is generated in response to shear stress cycles, following the cyclic-stress approach of Seed and Idriss. However, the model differs from traditional applications of this approach, where excess pore pressures are computed after a dynamic analysis, as a post-processing step. Instead, the model updates pore pressures continuously during a dynamic analysis, with reduction of effective stresses and shear strengths during shaking. The model has been updated to reflect changes in the simplified procedure, including revised correction factors for initial static stresses and a new shape for the cyclic strength curve. Other modifications include an effective-stress dependent elastic shear modulus, and a switch that prevents shear strength from dropping to residual strength during shaking. Adaptation of the model to 3-D analyses is also discussed. The performance of the model is illustrated with an analysis of the well known Lower San Fernando Dam case history.

INTRODUCTION

This paper reviews a practice-oriented constitutive model for analysis of liquefaction due to cyclic loading. The model consists of an elastic/plastic Mohr-Coulomb constitutive model, coupled to an empirical pore pressure generation procedure. Pore pressure is generated in response to shear stress cycles following the cyclic-stress approach of Seed and Idriss (1971). However, in contrast to the standard cyclic-stress approach, pore pressure is generated incrementally during shaking. Thus, pore pressure generation is fully integrated with a dynamic effective-stress analysis. This model has been implemented in the explicit finite-difference program FLAC (Itasca, 2008), as well as FLAC3D (Itasca, 2009).

The model was originally developed for analysis of Pleasant Valley Dam (Dames & Moore, 1985; Roth et al., 1991). Subsequently the model has been used for dynamic deformation analyses of other dams (Inel et al. 1993; Bureau et al., 1996, Dawson et al., 2001), dynamic soil-structure interaction of wharf structures (Roth et al., 1992) and prediction of dynamic centrifuge tests (Inel et al., 1993; Roth and Inel, 1993).

THE CYCLIC STRESS APPROACH

The cyclic-stress approach is the basis of the most widely used method for evaluating seismic liquefaction resistance, the quasi-empirical ‘Simplified Procedure’ (Seed & Idriss, 1971; Seed, 1979). In the cyclic-stress approach, the liquefaction potential of a soil layer is a function of the number and amplitude of shear stress cycles experienced during shaking. Shear stress cycles are measured in terms of the cyclic stress ratio (CSR), the ratio of cyclic shear stress τ_{cy} to initial vertical effective stress σ'_v .

$$CSR = \tau_{cy} / \sigma'_v$$

The cyclic shear stress τ_{cy} is defined as shown in Figure 1.

The liquefaction resistance of a soil is described by a cyclic strength curve, a plot of the number of cycles required for liquefaction to occur at a uniform cyclic stress ratio (Figure 1). The cyclic strength of a soil is a function of the relative density and fines content along with many other factors.

The cyclic-stress approach is traditionally applied in a post-processing mode. The time-history of shear stress for a soil layer is evaluated from a numerical analysis or by a simplified procedure. This time-history is then approximated as an equivalent number of uniform shear stress cycles N_{eq} . Finally, the number of equivalent cycles is compared to the soil’s cyclic strength curve to determine whether liquefaction would have been triggered by the equivalent number of cycles. In contrast, the model described here applies the cyclic-stress approach in an incremental form, one cycle of stress at a time.

PORE PRESSURE GENERATION MODEL

The pore-pressure generation model is built around the standard FLAC linear elastic/perfectly plastic Mohr-Coulomb model. The shear strength envelope is specified with a friction angle and a cohesion. The plastic flow rule can be associated or non-associated depending on the dilation angle specified. A yield cap is not defined, thus plastic volumetric strains are not predicted with the model. Unlike more physically realistic models in which pore pressure is generated through contraction of the soil skeleton, the present model bypasses the physical mechanism of liquefaction and generates pore pressure directly in response to shear stress cycles.

Pore Pressure Generation Procedure. The pore-pressure generation procedure is illustrated schematically in Figure 1. The shear stress time history of each element is monitored and shear stress cycles are counted. To follow as closely as possible the traditional cyclic-stress approach, the model monitors the shear stress on horizontal planes σ_{xy} rather than a shear stress invariant. As soon as a stress cycle is detected the excess pore pressure is incremented by an amount dependent on the cyclic stress ratio amplitude of that cycle. The generated excess pore pressure u_e is described in terms of the pore pressure ratio:

$$r_u = u_e / \sigma'_v$$

where σ'_v is the initial vertical effective stress.

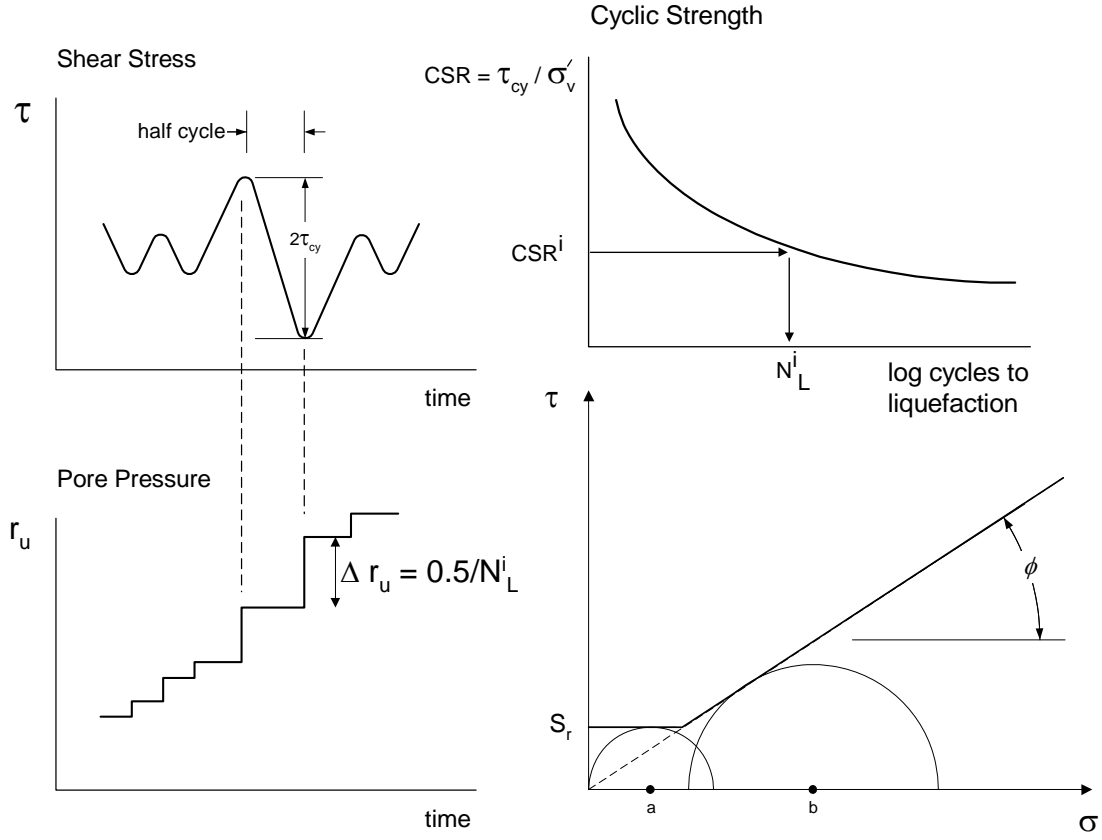


Figure 1. Pore pressure generation procedure, cyclic strength curve and shear strength.

In practice it is easier to count half cycles rather than full cycles. Half cycles are detected by looking for shear stress reversals (Figure 1). The cyclic stress τ_{cy} amplitude of each half cycle is half the difference between the preceding peak and valley.

Computation of an increment of excess pore pressure from the cyclic strength curve is also illustrated in Figure 1. If N_L^i uniform cycles are required for complete liquefaction triggering ($r_u = 1.0$) at a cyclic stress ratio CSR_i , then the increment in pore pressure ratio Δr_u^i for a half cycle is

$$\Delta r_u^i = 0.5 / N_L^i$$

and the increment in pore pressure u_e is then

$$\Delta u_e^i = \Delta r_u^i \sigma'_v$$

The effect of increasing the pore pressure is to decrease the effective stress and, thus, to decrease the shear strength. The shear modulus can also be decreased with pore pressure as described below in the Model Updates section. Even if the shear modulus is not reduced, decreasing the effective shear strength changes the resulting secant modulus and damping ratio for plastic stress-strain loops.

The Simplified Procedure contains correction factors for adjusting the cyclic strength for initial vertical effective stress greater than one atmosphere (K_σ) and for initial static shear stress (K_α). Both factors are included in the model.

Post-Liquefaction Residual Strength. The model incorporates the residual strength, S_r , of liquefied soils by using a two-segment failure envelope consisting of an initial segment defined by the residual cohesion value and a zero friction angle which extends to intersect the segment defined by the Mohr-Coulomb failure envelope (Figure 1). The post-liquefaction residual strength can be specified as either a constant value for a particular soil layer (Seed and Harder, 1990) or as a ratio (S_r / σ'_v) of the initial vertical effective stress (Olsen and Stark, 2002).

MODEL UPDATES

The model has been updated to reflect recent changes in the simplified procedure, including revised correction factors for initial static shear stresses and a new shape for the cyclic strength curve. Other modifications include an effective-stress dependent elastic shear modulus, and an optional switch that prevents shear strength from dropping to residual strength during shaking. Additional features were also required for adaptation of the model for use in the program FLAC^{3D}.

Revised Form for Cyclic Strength Curve. Previous versions of the model employed a bilinear cyclic strength curve. The cyclic strength curve now has the form:

$$CSR = CRR_{15} (N_L / 15)^{-1/B}$$

Where CRR_{15} and the constant B are input parameters. CRR_{15} is the cyclic stress ratio required for liquefaction in 15 cycles (corresponding to an earthquake magnitude of 7.5) while B is a constant controlling the overall slope of the cyclic strength curve. B defaults to 2.97, which corresponds to the magnitude scaling factors recommended by Idriss (1999) and Idriss and Boulanger (2008).

Reduction of Shear Modulus with Pore Pressure. In previous versions of the model, soil elastic constants remained constant during shaking. As an optional feature, the elastic shear modulus can now be reduced as pore pressure increases using the formula:

$$G = G_{initial} \sqrt{1 - r_u}$$

where r_u is the pore pressure ratio as defined above. The initial shear modulus $G_{initial}$ is stored during model initialization. The shear modulus obviously cannot be reduced all the way to zero, so the model includes an input parameter specifying the minimum allowable shear modulus for the element. This parameter defaults to 1 atmosphere.

Static Shear Stress Correction Factor K_α . The Simplified Procedure includes a correction factor, K_α , to account for a non-zero initial static shear stress. In previous version of the model this parameter could be input manually, but otherwise defaulted to 1.0 (no correction). In the current version of the model, K_α is computed following the procedure recommended by Idriss and Boulanger (2008), where K_α is a function of the static stress ratio α , the relative density, and the overburden stress σ'_v .

Overburden Correction Factor, K_σ . The correction factor for overburden stress, K_σ , is computed using the recommendations of the 1996 NCEER and 1998 NCEER/NSF workshops (Youd & Idriss, 2001) as follows.

$$K_\sigma = \left(\frac{\sigma'_v}{P_{atm}} \right)^{(f-1)}$$

where P_{atm} is atmospheric pressure. The parameter f is a function of the relative density D_r of the soil.

Maintaining Full Strength During Shaking. The model can now be run in a decoupled mode in which the excess pore pressure generated is disconnected from the effective stress, so that materials retain their full strength during shaking. This allows the model to be used in a manner more compatible with the traditional post-processing mode of the Simplified Procedure. In addition, scenarios can be investigated where the drop to residual strength does not occur until after shaking.

Adaptations for Use in 3-D Analyses. The model has also been implemented for use in the program FLAC^{3D}. Just as in the 2D model, stress cycles on the horizontal plane are counted. Rather than trying to incorporate cycle counting in multiple directions, the model monitors shear stress cycles in a single azimuth direction, specified by the user.

ANALYSIS OF LOWER SAN FERNANDO DAM

The pore-pressure generation model is demonstrated with an analysis of the Lower San Fernando Dam during the 1971 San Fernando earthquake, which triggered a large liquefaction-induced slide in the upstream slope. This near catastrophe has been studied extensively (e.g. Seed et al. 1973, 1975, 1988) and now serves as a classic case history on which many current ideas about liquefaction and post-liquefaction residual strength are based and tested.

The dam was 140 ft high at its maximum section. It was originally constructed by the hydraulic fill method, but rolled fill was added on several occasions to increase

reservoir capacity. The construction history and geotechnical properties of the dam materials are described by Seed et al. (1973, 1988). Detailed field investigations and trenching after the earthquake revealed that the slide was composed of large blocks of intact soil from the upstream slope riding over liquefied hydraulic fill. Slide material displaced as much as 200 ft into the reservoir and the dam crest dropped by as much as 40 ft.

The FLAC numerical model for the analysis is shown in Figure 2. Material properties used in the FLAC analysis, from Seed et al. (1973, 1988), are listed in Table 1. The four hydraulic fill layers were simulated with the pore-pressure generation model, while all other materials were simulated with a Mohr-Coulomb model. For the critical layer of hydraulic fill at the base of the dam, Seed (1988) suggested a blow count, $(N_1)_{60}$, of 11.5 for the upstream embankment and 12.5 for the downstream embankment. After fines correction for triggering, these become equivalent clean-sand values, $(N_1)_{60-cs}$, of 17.5 upstream and 18.5 downstream. The fines correction for post-liquefaction residual strength yields equivalent-clean sand values, $(N_1)_{60cs-Sr}$, of 13.5 upstream and 14.5 downstream. This base hydraulic fill layer was assigned a residual strength of 400 psf upstream and 450 psf downstream.

Table 1. Material Properties Used for Analysis of Lower San Fernando Dam.

Material	Saturated Unit Weight (lbs/ft)	Dry Unit Weight (lbs/ft)	K_{2max}	Friction Angle (degrees)	$(N_1)_{60-cs}$ Upstream / Downstream	Residual Strength (psf)
Hydraulic Fill 1 Elev. 1000–1023	126	106	43	37	17.5 / 18.5	400/450
Hydraulic Fill 2 Elev. 1023–1038	126	106	43	37	30 / 30	1600
Hydraulic Fill 3 Elev. 1038–1056	126	106	43	37	17.5 / 18.5	400/450
Hydraulic Fill 4 Elev. 1056–1074	126	106	43	37	25 / 25	1600
Ground Shale	130	106	52	37	-	-
Rolled Fill	140	125	55	37	-	-
Clay Core	126	106	$G = 700 s_u$	$s_u = 2000$ psf	-	-
Upper Alluvium	130	110	52	37	-	-
Lower Alluvium	130	110	105	37	-	-
Bedrock	140	135	$v_s = 3600$ ft/sec	-	-	-

Notes: G : shear modulus, s_u : undrained shear strength, v_s : shear wave velocity

Dynamic analysis was performed using the ‘Modified Pacoima Record’, an acceleration time history adapted by Seed et al. (1973) from a rock outcrop motion at the nearby Pacoima Dam. This record, shown in Figure 2, has a peak ground acceleration (PGA) of 0.6g with approximately 10 seconds of strong motion. The motion was applied to the model through a compliant (non reflective) base.

Time histories of pore pressure ratio for four points within the critical hydraulic fill layer at the base of the dam are shown in Figure 2. These display the

typical stair-step, monotonically increasing behavior produced by this pore pressure generation model. The detailed behavior of the pore-pressure generation model for point 2 is shown in Figure 3. Contours of pore pressure ratio after shaking (Figure 4)

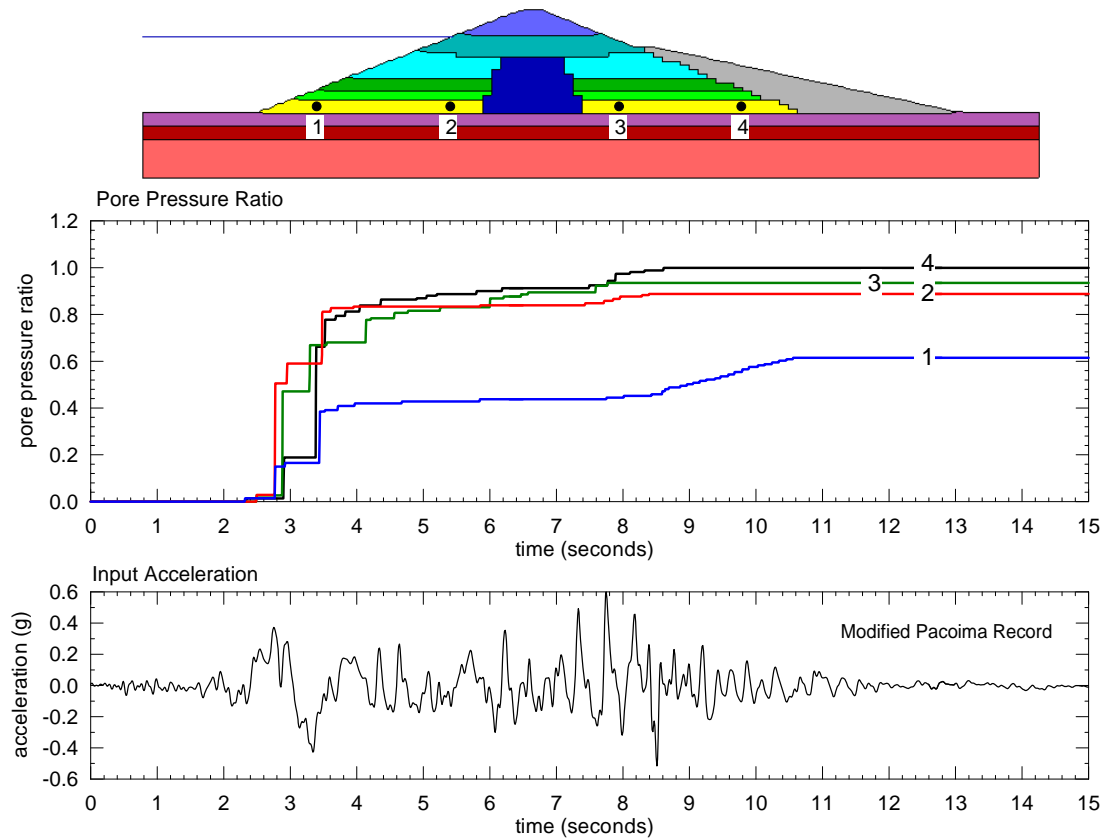


Figure 2. Pore pressure ratio versus time for points within critical hydraulic fill layer at base of dam.

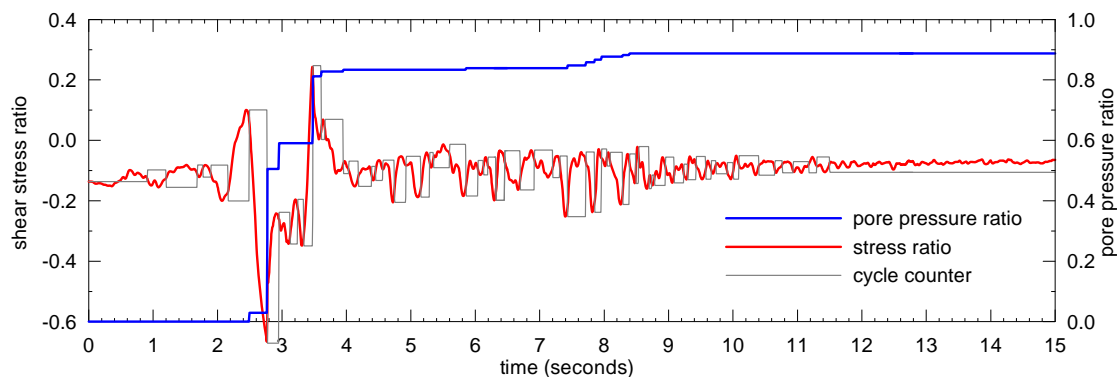


Figure 3. Time history of stress ratio and cycle counter for Point 2 (Figure 2), along with the pore pressure generated (right axis).

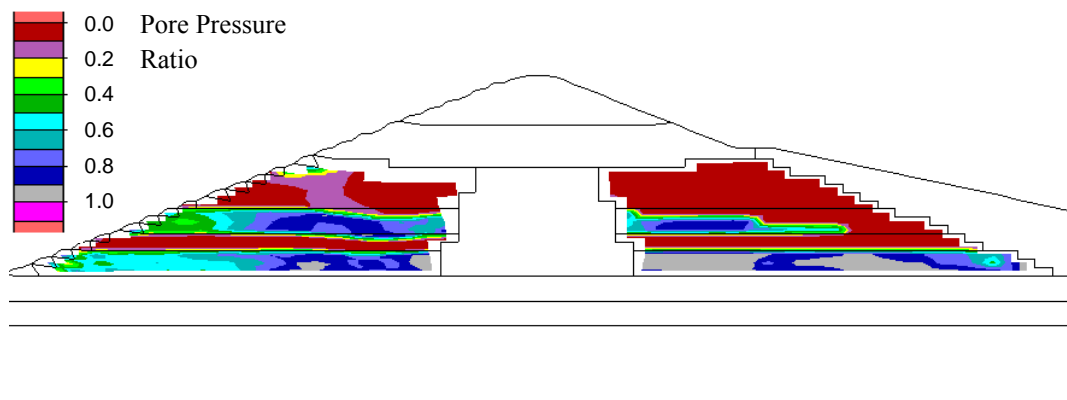


Figure 4. Contours of pore pressure ratio after shaking.

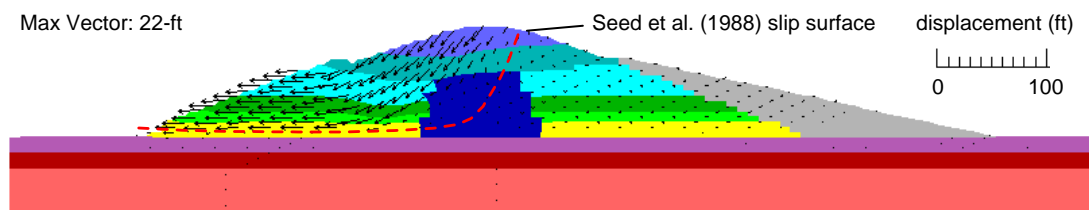


Figure 5. Displacement vectors at 30 seconds.

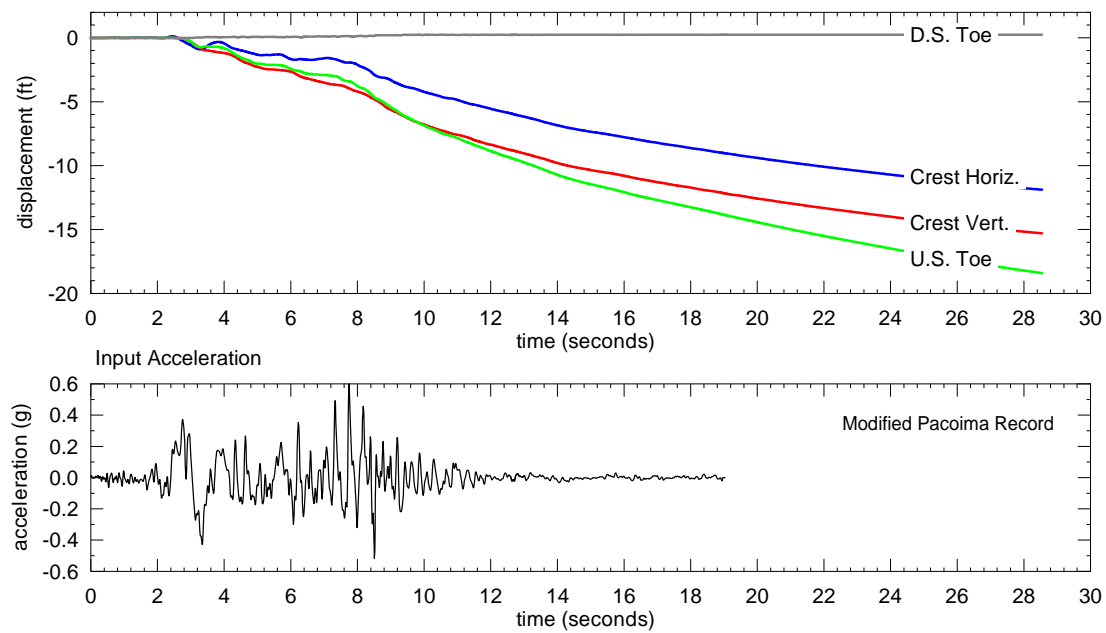


Figure 6. Time histories of displacement at dam crest and upstream and downstream toes.

show extensive liquefaction not only in the upstream shell, but also in the downstream shell. It can be seen from Figure 5 that a large slide occurs in the upstream slope, while the downstream slope moves very little. This figure (5) shows displacement vectors at approximately 30 seconds into the analysis, after which the numerical mesh became too distorted to continue the analysis. At this point the dam crest has dropped 15 ft and moved upstream horizontally 12 ft. The upstream toe has moved 18 feet horizontally, while the downstream toe has moved only 0.25 ft. Figure 6 shows time histories of these displacements.

CONCLUSION

An update has been described of a simple, practice-oriented, cycle-counting liquefaction model that has been in use since 1985. The update included revised correction factors for initial static stresses and a new shape for the cyclic strength curve. Other modifications included an effective-stress dependent elastic shear modulus, and a switch that prevents shear strength from dropping to residual strength during shaking. Adaptation of the model to 3-D analyses was also discussed. The performance of the model was illustrated with an analysis of the well known Lower San Fernando Dam case history.

ACKNOWLEDGEMENTS

The authors would like to acknowledge helpful discussions held with Mike Beaty, David Serafini and Vlad Perlea on the pore pressure generation model and its application to the Lower San Fernando Dam case history.

REFERENCES

- Bureau, G., Inel, S., Davis, C. A., and Roth, W. H. (1996) "Seismic Response of Los Angeles Dam, CA, during the 1994 Northridge Earthquake." *Proc. USCOLD 1996 Annual Meeting*, Los Angeles, California. 281-295.
- Dames and Moore, (1985) *Evaluations of earthquake-induced deformations of Pleasant Valley Dam*, Report to City of Los Angeles, Dept. of Water and Power, May 24.
- Dawson, E. M., Roth, W. H., Nesarajah, S., Bureau, G. and Davis, C. A. (2001) "A practice-oriented pore pressure generation model." *Proceedings, 2nd FLAC Symposium on Numerical Modeling in Geomechanics*, Oct. 29-31, Lyon, France.
- Idriss, I. M. (1999) "An update to the Seed-Idriss simplified procedure for evaluating liquefaction potential." in *Proceedings, TRB Workshop on New Approaches to Liquefaction*, Publication No. FHWA-RD-99-165, Federal Highway Administration, January.
- Idriss, I. M. and Boulanger, R. W. (2008) *Soil Liquefaction During Earthquakes*, Monograph MNO-12, Earthquake Engineering Research Institute, Oakland CA.

- Inel, S., Roth, W. H., and de Rubertis, C. (1993) "Nonlinear Dynamic Effective-Stress Analysis of Two Case Histories." *3rd Int. Conf. on Case Histories in Geotechnical Engineering, St. Louis, Missouri*. Paper No. 14.14: 1735-1741.
- Itasca Consulting Group, Inc. (2008) *FLAC – Fast Lagrangian Analysis of Continua, Ver. 6.0 User's Manual*. Minneapolis: Itasca.
- Itasca Consulting Group, Inc. (2009) *FLAC3D – Fast Lagrangian Analysis of Continua in 3 Dimensions, Ver. 4.0 User's Manual*. Minneapolis: Itasca.
- Olsen, S. M. and Stark, T. D. (2002) "Liquefied Strength ratio from liquefaction flow case histories." *Canadian Geotechnical J.* **39**, 629-47.
- Roth, W. H., Bureau, G., and Brodt, G. (1991) "Pleasant Valley Dam: An Approach to Quantifying the Effect of Foundation Liquefaction." *17th International Congress on Large Dams, Vienna*. 1199-1223.
- Roth, W. H., Fong, H., and de Rubertis C. (1992) "Batter Piles and the Seismic Performance of Pile-Supported Wharves." *Proceedings, ASCE Specialty Conference, Ports '92, Seattle, Washington*. 336-349.
- Roth, W. H. and Inel, S. (1993) "An Engineering Approach to the Analysis of VELACS Centrifuge Tests." *International Conference on the Verification of Numerical Procedures for the Analysis of Soil Liquefaction Problems*, Davis, California. Arulanandan and Scott (eds), Balkema, Rotterdam, 1209-1216.
- Roth, W. H., Inel, S., Davis, C., and Brodt, G. (1993) "Upper San Fernando Dam 1971 Revisited." *10th Annual Conference of the Association of State Dam Safety Officials, Kansas City, Missouri*
- Seed, H. B. and Idriss, I. M. (1971) "Simplified Procedure for evaluating soil liquefaction potential." *J. Soil Mech. And Found. Div. ASCE*, 97(SM9): 1249-1273.
- Seed, H. B., Lee, K. L., Idriss, I. M. and Makdisi, F. I. (1973) *Analysis of the slides in the San Fernando dams during the earthquake of February 9, 1971*, Rep. No. UCB/EERC-72/02, Univ. of California Berkeley.
- Seed, H. B., Lee, K. L., Idriss, I. M. and Makdisi, F. I. (1975) "The slides in the San Fernando Dams during the earthquake of February 9, 1971." *J. Geotech. Eng. Div. ASCE*, 101(7): 651-688.
- Seed, H. B. (1979) "Soil liquefaction and cyclic mobility evaluation for level ground during earthquakes." *J. Geotech. Eng. Div., ASCE*, 105(GT2): 201-255.
- Seed, H. B., Seed, R. B., Harder, L. F. and Jong, H. L. (1988) *Re-evaluation of the slide in the Lower San Fernando Dam in the 1971 San Fernando Earthquake*, Rep. No. UCB/EERC-88/04, Univ. of California Berkeley.
- Seed, R. B. and Harder, L. F. (1990) "SPT-Based Analysis of Cyclic Pore Pressure Generation and Undrained Residual Strength." in., *Proceedings, H. Bolton Seed Memorial Symposium*, J.M Duncan (ed.), University of California, Berkeley, Vol. 2. 351-376.
- Youd, T. L. and Idriss, I. M. (2001) "Liquefaction resistance of soils: summary report from the 1996 NCEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils." *J. Geotech. and Geoenviron. Eng., ASCE*, 127(4): 297-313.